1. 請證明愛因斯坦之比熱定律預測低溫下比熱趨近於零，高溫下趨近於古典的結果。

$$C_{v,m} = 3R\,f^2$$
$$f = \frac{\theta_E}{T}\left(\frac{e^{\theta_E/2T}}{e^{\theta_E/T}-1}\right)$$
$$\theta_E = \frac{h\nu}{k}$$

低溫下，T 趨近於 0，因為$e^{\frac{\theta E}{T}}$很大可以忽略-1，

$f \approx \frac{\theta E}{T}\left(\frac{e^{\frac{\theta E}{2T}}}{e^{\frac{\theta E}{T}}}\right) \approx \frac{\theta E}{T}e^{\left(\frac{\theta E}{2T}-\frac{\theta E}{2T}\right)} \approx \frac{\theta E}{T}e^{\left(\frac{-\theta E}{2T}\right)}$，T→ 0, $e^{\left(\frac{-\theta E}{2T}\right)}$下降的趨勢，比$\frac{\theta E}{T}$升高的趨勢還要快，$e^{\left(\frac{-\theta E}{2T}\right)} \to 0, f = 0$, $C_{v,m} = 0$ 所以低溫下，Cv,m=3Rf^2=3R $\times 0^2$=0

高溫下，T 趨近於∞，使用 Tayler expansion 把f展開然後取到一階項(因為越高次項，T 只會越小)，$f \approx \frac{\theta E}{T}\frac{1+\frac{\theta E}{2T}}{1+\frac{\theta E}{T}-1} \approx \frac{\theta E}{T}\frac{1+\frac{\theta E}{2T}}{\frac{\theta E}{T}} = \frac{\theta E}{T}\frac{1}{\frac{\theta E}{T}} = 1$，所以高溫下 Cv,m=3R$f^2$=3R $\times 1^2$=3R

2. 請證明波爾氫原子模型可解釋雷德堡公式的由來。

波爾堆導出氫原子的能量 ： $E_n = \frac{-Z^2e^2}{8\pi\varepsilon_0}\frac{1}{n^2}, a_0 = \frac{4\pi\varepsilon_0\hbar^2}{me^2}$

$\Delta\ E_n = \frac{-Z^2e^2}{8\pi\varepsilon_0 a_0}\left(\frac{1}{{n_f}^2}-\frac{1}{{n_i}^2}\right) = \frac{-Z^2e^2}{8\pi\varepsilon_0}\frac{me^2}{4\pi\varepsilon_0\hbar}\left(\frac{1}{{n_f}^2}-\frac{1}{{n_i}^2}\right)$, z=1, $\hbar = \frac{h}{2\pi}$

$\Delta\ E_n = \frac{m\ \ e^2}{8{\varepsilon_0}^2h^2}\left(\frac{1}{{n_i}^2}-\frac{1}{{n_f}^2}\right) = hc\tilde{v},\ \ \tilde{v} = \frac{m\ \ e^2}{8{\varepsilon_0}^2h^3C}\left(\frac{1}{{n_i}^2}-\frac{1}{{n_f}^2}\right)$