O S O S
G T G T
5 E 5 E
N O N O
O S O S
G T G T
5 E 5 E
N O N O
O S O S
G T G T
5 E 5 E

09/22

(1) $y' = \frac{1}{2}y_0$，代入 $y' = y_0 \cdot e^{-kt'}$

$\frac{1}{2}y_0 = y_0 \cdot e^{t_{1/2} \cdot K}$，$\frac{1}{2} = e^{(5715yr)K}$

$\ln\frac{1}{2} = (5715yr)K$，$K = -1.21 \times 10^{-4}$

$$y' = y_0 e^{(-1.21\times10^{-4})\times t'}$$

$$y' = y_0 e^{(-1.21\times10^{-4})\times 4000}$$

$$\frac{y'}{y_0} = 0.6163 \doteqdot \underline{61.6\%}$$ #

(2) $(x-y)(dx-dy)=0$

移項 $(x-y)\,dx+(y-x)\,dy=0$

Step 1: 檢查ODE

令 $M=x-y$

$N=y-x$

$$\frac{\partial M}{\partial y}=\frac{\partial(x-y)}{\partial y}=-1,\quad \frac{\partial N}{\partial x}=\frac{\partial(y-x)}{\partial x}=-1$$

$\because \frac{\partial M}{\partial y}=\frac{\partial N}{\partial x}=-1$ $\therefore$ this equation is exact ODE.

Step 2: 求 general solution

令 general solution. $u(x,y)=c$

$$u=\int M\,dx+k(y)=\int(x-y)\,dx+k(y)$$
$$=\frac{1}{2}x^2-xy+k(y)$$

$$\frac{\partial u}{\partial y}=\frac{\partial\left(\frac{1}{2}x^2-xy+k(y)\right)}{\partial y}=-x+\frac{dk(y)}{dy}=y-x$$

得 $\frac{dk(y)}{dy}=y$, $k(y)=\int y\,dy=\frac{1}{2}y^2$, general solution $u(x,y)=\frac{1}{2}x^2-xy+\frac{1}{2}y^2$ ※