

Quantum Chemistry and Molecular Processes

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This article gives an overview of modern electronic structure theory, which is the development of approximate quantum mechanical methods for calculating the ground and excited electronic states of molecules. A series of seven topics are discussed which have contributed to the present state of the field and that illustrate in a general way some of the essential physical models and approximations that underpin electronic structure calculations. These topics begin from the definition of theoretical model chemistries and the treatment of electron correlation by wave function-based techniques and density functional theory, for both molecular energies and a range of molecular properties. Beyond these main theoretical issues, questions of chemical interpretation, computational algorithms, and the modeling of surrounding environment are also discussed. Collectively these topics define many of the classes of chemical problems which can be reliably and feasibly solved by such calculations at present, and conversely they also serve to define a number of unresolved challenges for the future.

1. Introduction

In this article, I will give a general, nonmathematical view of the past, present, and future of molecular electronic structure theory (which is synonymously termed quantum chemistry or a branch of molecular quantum mechanics) and its connection to current problems in chemistry at large. To do this in a way which is neither encyclopedic or excessively superficial, I choose to focus on seven different topics where critical advances have occurred or are occurring. In many cases, these areas also reveal issues which are not yet adequately solved or where new problems that must be addressed in the future are arising. Unavoidably, this requires omission of some worthy topics and also omission of innumerable possible literature citations! In a number of topics there are rich interconnections to other, more specialized, articles in this Centennial Issue, which I will point out where appropriate.

Electronic structure theory for the purposes of this discussion is the application of the principles of quantum mechanics to calculating the stationary states of molecules and, to a lesser extent, characterizing transitions between these states. Shortly after the discovery of quantum mechanics, Dirac declared¹ that “the underlying physical laws necessary for the mathematical theory of a large part of physics and the whole of chemistry are completely known”. A few prominent chemists soon agreed, with Lewis stating in 1933² that “there can be no question that in the Schrödinger equation we very nearly have the mathematical foundation for the solution of the whole problem of atomic and molecular structure”, although he cautioned that “the problem of the many bodies contained in the atom and the molecule cannot be completely solved without a great further development in mathematical technique”. This paper is a broad overview of where we stand in regard to fulfilling or contradicting these statements, some 60 years later, and where we may be heading.

As will be evident to anyone skimming the contents of an issue of *The Journal of Physical Chemistry*, in this, its 100th year, the impact of molecular calculations employing electronic structure theory methods on chemistry is now significant. Many

experimental groups routinely employ *ab initio* (meaning “from first principles”, an adjective whose accuracy will hopefully become clear as the article progresses) molecular orbital calculations as a valuable adjunct to experiment. Such calculations may yield a prediction of the molecular structure for a proposed intermediate, or even a sequence of intermediates which together define a proposed reaction path. In terms of being a tool which is increasingly used by nonexperts, this field is rightfully on the way to following branches of spectroscopy in becoming another weapon in the practicing chemist’s armory. Several interesting chemical case histories^{3,4} detail the evolution of electronic structure methods toward the goal of quantitatively predicting molecular structure and reactivity.

Can you, the practicing chemist, indeed employ these techniques in a robust and reliable fashion to solve chemical problems? To the extent you can, then what are the unsolved issues in the field, or should it be considered a mature area where few fundamental challenges remain? To the extent you cannot, then what of significance has been accomplished over the past several decades in this subject? I will discuss the current state of this field, in terms of both accomplishments and unresolved problems and open issues, in the context of a series of seven topics which I believe have played or are playing central roles in its development. The ordering is chosen to be pedagogical rather than chronological or any other ranking.

In section 2, I discuss theoretical model chemistries. The general problem of electronic structure theory is how to apply the principles of quantum mechanics to molecular problems, when we cannot solve the underlying equations exactly. In fact, absolute errors associated with common approximations can be large relative to chemical energy changes, and we must often rely on substantial cancellation of errors in order to achieve useful chemical results. A theoretical model chemistry is a set of approximations sufficient to calculate all observable properties of a molecular system. This concept is very useful for guiding the development of new theories with broad chemical applicability, as we can delineate properties of approximate theories which are essential for this purpose. In this context, we discuss the simplest theoretical model in which each electron moves in the average field of all others. The resulting

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probability distributions are the molecular orbitals of Hartree–Fock theory.

The difference between the Hartree–Fock (HF) energy and the exact energy is usually termed the electron correlation energy, since it is due to detailed *correlations* between electrons which are averaged out in the HF approach. Approximating the correlation energy is essential for reliably calculating fundamental properties such as bond energies. In section 3, I discuss two general approaches to the correlation problem. The first is based on the HF molecular orbitals and is termed “single reference (SR) methods”, since the occupied HF orbitals comprise a wave function which is a single determinant. The second approach, multireference (MR) methods, seeks orbitals which are appropriate to multideterminantal wave functions. SR and MR methods have strengths and weaknesses which are somewhat complementary. They are discussed in the context of theoretical model chemistries and obtaining potential energy surfaces including bond-breaking processes. This section connects closely to the article by Raghavachari and Anderson in this issue,⁵ which provides an in-depth discussion of electron correlation.

The correlation methods discussed above represent one main branch of electronic structure theory, namely, that based on seeking tractable approximations to the exact *wave function*. However, it has been known for three decades that the exact energy is in fact a functional of only the electron *density* (a function of only 3, rather than $3n$, variables). The only catch is that the functional is not known. In section 4, I discuss density functional theory (DFT) which focuses on the direct determination of the electron density, without the intermediary of a wave function, by developing approximate functional forms. DFT is an increasingly significant branch of molecular quantum mechanics with its primary virtue being the fact that it can be applied to larger chemical systems than the sophisticated electron correlation techniques discussed above. A more technical and comprehensive summary of the state of the art can be found in the article by Kohn et al. in this issue.⁶

To this point, I will have briefly reviewed the methods that yield an approximate molecular energy for a given nuclear arrangement. In section 5, the focus is the use of molecular electronic structure methods to obtain molecular properties other than the energy. This includes molecular structure, vibrational frequencies, permanent and induced electrical moments, and even electronic transitions, just to name a few examples. The success of electronic structure methods in predicting a large number of important molecular properties has been an important part of their emergence as a legitimate tool for many chemical problems. Often the accuracy obtained for molecular properties is significantly higher than might be expected based on the performance of the corresponding theoretical method for relative energies alone. This section is partly intended to give some perspective on what can and cannot be adequately calculated by electronic structure methods at present.

It is perhaps self-evident that the objective in developing new electronic structure theories is to strike improved tradeoffs between accuracy and feasibility. That is after all what different theoretical model chemistries represent. However, due to the complexity of electronic structure methods as problems coupling quantum mechanics, applied mathematics, and scientific computing, there has been and still is tremendous progress occurring in the algorithms used to implement given theoretical model chemistries. I discuss this area in general terms in section 6, since an overall understanding of the computational requirements of different theoretical models leads to an overall understanding of the range of molecule sizes to which they can be applied. It

remains a crucial problem to make the methods whose veracity has been established on small molecules applicable to larger systems. This point will be illustrated by some rough estimates on the size of molecules which may be amenable to electronic structure calculations by early in the next century.

Up to this stage in the article, I have been implicitly considering the problem of solving for the electronic structure of an isolated molecule, as is appropriate for chemical problems occurring in the gas phase under collision-free conditions. This is scarcely the only important environment in which chemical processes occur, although it does have the merit of being simplest to model! In section 7, I give an overview of the ways in which environments such as those in solution, or for molecules interacting with extended surfaces, are currently treated. This is an area where significant progress is likely to occur in the future, probably in the context of closer connections between electronic structure methods and other areas of theoretical chemistry and solid state physics.

The last topic I consider involves trying to close the circle that opened when quantum mechanics first offered the possibility of unbiased calculations of the properties of molecules. Namely, how should the results of such calculations be interpreted to yield chemical insight and to influence our paradigms of structure, bonding, and reactivity? This is a difficult and open-ended subject where there has been some progress and considerable controversy. I give my perspective on this area in section 8, noting that many of the standard theoretical model chemistries do not easily yield insights into chemical bonding from their intricate wave functions. There is hope for a closer convergence in the future, because the needs for improvements in algorithms discussed in section 6, and the modeling of the environment in section 7, connect intimately to concepts of localization of electrons in molecules, which also lies at the heart of most ideas of chemical bonding.

Finally, I close with a brief summary of the present status of the field and some of the key unresolved issues for the future. There has been impressive progress made to this point, which is now reflected in the acceptance of electronic structure calculations from the general chemical community. In my opinion, this present state coupled with significant unresolved challenges makes the future of molecular electronic structure theory seem bright and dynamic indeed.

2. Theoretical Model Chemistries

The objective of ab initio electronic structure theory is exact solution of the time-independent Schrödinger equation⁷

$$H(\mathbf{r};\mathbf{R}) \Psi(\mathbf{r};\mathbf{R}) = E(\mathbf{R}) \Psi(\mathbf{r};\mathbf{R})$$

This yields first the molecular wavefunction, $\Psi(\mathbf{r};\mathbf{R})$, which depends explicitly on the $3n$ coordinates of all n electrons, collectively denoted as \mathbf{r} , and implicitly on the nuclear coordinates also, collectively denoted as \mathbf{R} . The implicit rather than explicit nuclear coordinate dependence is because we have made the Born–Oppenheimer separation⁸ of (slow) nuclear motion from (fast) electronic motion. We also obtain the molecular energy, $E(\mathbf{R})$, which depends parametrically on the nuclear positions. Hence, $E(\mathbf{R})$ defines the potential energy surface (PES) obtained from solution of the Schrödinger equation subject to the Born–Oppenheimer approximation. H is the molecular electronic Hamiltonian (or total energy) operator, subject to frozen nuclei, consisting of kinetic energy, electron–nuclear attraction, electron–electron repulsion, and nuclear–nuclear repulsion. In atomic units, its explicit form is

$$H(r;R) = -\frac{1}{2}\sum_i^n \left(\frac{\partial^2}{\partial x_i^2} + \frac{\partial^2}{\partial y_i^2} + \frac{\partial^2}{\partial z_i^2} \right) - \sum_i^n \sum_\alpha^A \frac{Z_\alpha}{|\mathbf{r}_i - \mathbf{R}_\alpha|} + \frac{1}{2}\sum_i^n \sum_j^n \frac{1}{|\mathbf{r}_i - \mathbf{r}_j|} + \frac{1}{2}\sum_\alpha^A \sum_\beta^A \frac{Z_\alpha Z_\beta}{|\mathbf{R}_\alpha - \mathbf{R}_\beta|}$$

Partial differential equations in $3n$ unknowns such as we have here are completely intractable to solve exactly, and therefore two important approximations are usually made. Together these approximations will determine how closely a given calculation will approach the exact result.

The first approximation is that we do not attempt to solve the exact equations. Instead, we solve related, but simpler, sets of equations. For example, it is desirable to reduce the function of $3n$ variables to n functions (orbitals), each depending only on three variables. Each such molecular orbital (MO) would describe the probability distribution of a single electron and would be determined by the electron moving in the average field of all other electrons. Enforcing Fermion antisymmetry leads us to construct a trial wave function which is a determinant of molecular orbitals and obtain the optimum MO's by variationally minimizing the energy. This is the much used (and sometimes abused!) Hartree–Fock (HF) approximation,^{9,10} which is the foundation of much of modern molecular orbital theory.

The HF method is simply a mean field method applied to the many-electron problem,^{11–13} and for this reason the HF solution is often called the self-consistent field (SCF). As we shall see in section 4, density functional theories are in a sense improved SCF methods. Approximations to the many-electron wave function that go beyond the HF level must include details of the instantaneous electron–electron correlations, which are averaged out in the SCF. For this reason, in HF calculations, electrons approach each other a little too closely, and the HF wave function is slightly “too ionic” in character. Calculations which include electron–electron correlation can correct this deficiency and are discussed in section 3.

The second approximation which is usually invoked is that we solve the inexact equations (for example, the Hartree–Fock equations) inexactly. Because computers are far better at solving algebraic sets of equations rather than differential sets of equations, it is advantageous to convert the HF equations into a problem in linear algebra by expanding the unknown molecular orbital functions in terms of a given, fixed set of functions, whose number is finite.¹⁴ If the functions are wisely chosen, then as their number increases, increasingly accurate approximate MO's will be obtained. These functions are usually called the atomic orbital (AO) basis, because they are atom-centered and resemble solutions to the HF problem for the constituent atoms of a molecule. Standardized basis sets are now available going under many different, seemingly obscure acronyms, and detailed reviews^{15,16} are available to help guide both the uninitiated and the initiated through this aspect of an electronic structure calculation.

AO basis functions are Gaussian functions, or linear combinations of Gaussians, since this form permits all required matrix elements to be evaluated analytically.¹⁷ The simplest widely used basis sets are of the (aptly named) “minimal” type (one AO per valence atomic orbital, or “single zeta”), while more flexible sets are the “split valence” or “double zeta” type (two AO's per valence atomic orbital). It is possible to proceed to triple zeta and onward. The physical purpose of providing multiple basis functions per atomic orbital is to allow the size of orbitals to increase (for example, along a bond axis) or

FCI	FCI/ STO-3G	FCI/ 3-21G	FCI/ 6-31G*	FCI/ 6-311G(2df)		exact
...						
CCSD(T)	CCSD(T)/ STO-3G	CCSD(T)/ 3-21G	CCSD(T)/ 6-31G*	CCSD(T)/ 6-311G(2df)		CCSD(T) limit
CCSD	CCSD/ STO-3G	CCSD/ 3-21G	CCSD/ 6-31G*	CCSD/ 6-311G(2df)		CCSD limit
MP2	MP2/ STO-3G	MP2/ 3-21G	MP2/ 6-31G*	MP2/ 6-311G(2df)		MP2 limit
HF	HF/ STO-3G	HF/ 3-21G	HF/ 6-31G*	HF/ 6-311G(2df)		HF limit
	STO-3G	3-21G	6-31G*	6-311G(2df)	...	complete

↑ Correlation → AO Basis Set

Figure 1. A theoretical model chemistry is defined by the two main approximations that are made in order to make solution of the time-independent Schrödinger equation tractable. These are the level of correlation treatment and the extent of completeness of the set of basis functions which are used to represent the molecular orbitals. The chart arranges approximations in order of increasing accuracy along the two axes, so that model chemistries toward the top right approach exact solution of the Schrödinger equation. The horizontal axis contains a series of basis sets of increasing quality (minimal, double zeta, double zeta plus polarization, triple zeta plus double polarization functions, etc.; see ref 20 for details) as discussed in section 2. The vertical axis contains a series of increasingly sophisticated models of electron correlation, as discussed in section 3. The accuracy of these methods for relative energies is discussed in section 3 and for the calculation of molecular properties such as geometries and vibrational frequencies in section 5. State-of-the-art density functional theory methods as discussed in section 4 lie roughly at the MP2 level of accuracy.

diminish (for example, perpendicular to a bond axis). It is also common to enhance double or triple zeta basis sets with polarization functions (one or more sets of d functions on first-row atoms, making basis sets of the form “DZP” or “TZ2P”, for example) to describe small displacements of the orbitals from their atomic centers in the molecular environment and for the description of electron correlation via the methods of section 3. Finally, for anions and Rydberg excited states, additional diffuse functions are necessary.

The combination of the first approximation, which is to solve a set of approximate equations, rather than the exact Schrödinger equation, and the second approximation, which is the use of an incomplete set of expansion functions for the MO's, constitutes (at least in principle!) the sole sources of error in an ab initio electronic structure calculation. All approximations can therefore be conveniently represented on a two-dimensional chart,¹⁸ where the two axes correspond to the correlation treatment and the basis set. Such a chart is given in Figure 1 with a series of standard approximations listed on the two axes. The basis set entries on the horizontal axis were described above. We shall discuss the as-yet-undefined entries on the vertical axis in section 3, as they represent increasing accuracy correlation treatments. An exact correlation treatment with an infinite (complete) basis set constitutes a restatement of the Schrödinger equation.

So, what is a theoretical model chemistry, and how does it connect to these two approximations and the diagram embodied in Figure 1? Any entry in Figure 1 constitutes, in principle, a method which can be applied to any molecule, to determine its properties such as structure, energetics, etc. Thus, if I apply a given method, say HF, with a given basis set (for instance, the

minimal STO-3G basis) to a range of molecules, I can determine all properties without further input. This pair of approximations defines a theoretical model chemistry,¹⁹ which for my present example would be conventionally denoted²⁰ as HF/STO-3G. If a given theoretical model chemistry is applied to a wide range of phenomena and it proves capable of accurately representing real chemistry, then it acquires some predictive value for studying systems that are not yet well characterized. More realistically, a given theoretical model achieves different levels of success on different classes of problems, and the process of model validation serves to distinguish these categories to permit reliable application of the method to at least a limited range of problems.

This approach to studying molecular properties at a fixed level of approximation, for example HF/STO-3G, across a wide range of molecules, stands in contrast to the possible alternative approach of attempting to calculate the most accurate solution possible for a single molecule of interest, perhaps by demonstrating convergence of the desired property with respect to improvements in the basis set and correlation treatment. The latter approach, which could be characterized as a “molecule at a time” philosophy, makes it difficult to compare results between different molecules that are not treated at a uniform level of approximation.

The notion of a theoretical model chemistry can act as a useful guide to those properties of the exact solution which would be most valuable to preserve in approximate schemes. For example, if we wish to calculate the dissociation energy of a molecule into its two fragments, it is immediately obvious that we require approximate methods where the accuracy of the calculation does not degrade as the molecule gets larger. More mathematically, the energy should scale extensively with the number of electrons²¹ (“size extensivity”), or essentially equivalently, the energy of a system of noninteracting fragments should be exactly the sum of separate calculations on the fragments (“size consistency”).¹⁹ This size-scaling property is satisfied for Hartree–Fock theory, but not for many approximate configuration interaction methods of electron correlation.

For unambiguous comparison of calculations on different molecules, a theoretical model chemistry should be completely defined once the number of electrons and the nuclei and their positions are specified (thereby defining the molecular Hamiltonian). Not all widely used methods satisfy this property. Another property that is also desirable in the context of a theoretical model chemistry is that the method yield continuous potential energy surfaces. Like size scaling, this seems obvious to state, but a variety of widely applied approximations based on configuration selection fail to satisfy this criterion. Other criteria for a successful theoretical model chemistry include sufficient accuracy and low enough computational cost for wide application.

The successful application of electronic structure methods to wide classes of chemical problems reduces to the selection of a theoretical model chemistry which adequately balances accuracy (how closely is the exact result approached?) and feasibility (the most accurate methods are far more computationally demanding than the least accurate and therefore, for fixed computational resources, can be applied only to far smaller molecules). A principal achievement of applied electronic structure theory is delineating the precision expected from a given theoretical model for various classes of molecules and properties. We will roughly indicate during the balance of this article what can be achieved with some of the standard electronic structure methods. More detailed information is of course available in the review literature²² and various monographs.^{23,24}

3. Chemical Accuracy and Electron Correlations

The differences between Hartree–Fock molecular energies and our best estimates of exact nonrelativistic energies are very small in percentage terms, typically being on the order of only a percent or so for first-row atoms, for example.²⁵ This energy difference is termed the electron correlation energy. The energy changes in reactive chemical processes are also very small fractions of the total absolute energy of the molecules that participate in them, since usually only a small number of electrons are involved in bond making and breaking, and furthermore these active electrons are always the least tightly bound. In fact, the absolute magnitude of the electron correlation energy per electron pair is in the vicinity of 50–100 kJ/mol, which is exactly in the region we care about in the context of chemical transformations. Furthermore, the electron correlation energy can also change quite significantly in many elementary chemical processes, such as the splitting of an electron pair bond in a dissociation, which clearly takes the correlation energy of that pair of electrons to zero.

Given general considerations such as these, it is not surprising that theoretical model chemistries based on the Hartree–Fock method were not uniformly successful for calculations of bond dissociation energies and the relative energies of different isomeric structures. Success depends critically on cancellation of errors (the neglected correlation energy) between reactants and products! Hence, processes such as internal rotation and energy changes for so-called isodesmic reactions where the number of electron pairs was conserved²⁰ are treated most reliably by the HF method. As discussed in section 5, the greatest successes of the HF method are in prediction of equilibrium structures and properties other than relative energies. To achieve the target accuracy (4–8 kJ/mol) considered necessary for electronic structure calculations of relative energies to be chemically useful would require near quantitative descriptions of electron correlation. The resulting methods then fill out the spectrum of beyond-HF methods in Figure 1.

Let us define the exact correlation energy, given an AO basis set. From solving the HF equations, we have a set of occupied MO's, comprising the HF determinant, and a set of unoccupied MO's. The exact many-Fermion wave function describing a molecule must be antisymmetric just like the HF determinant. Therefore, it must be representable as a mixture or linear combination of determinants. If we mix together all possible determinants (i.e., all possible orbital occupancies) and variationally determine the mixing coefficients, that will yield us the exact solution to eq 1 within the chosen AO basis. This is “full configuration interaction” (FCI), which represents the exact description of electron correlation within a chosen basis. FCI is computationally unfeasible beyond systems of just several atoms,²⁶ so we must seek approximations.

The simplest treatment of electron correlation that has been of general use as a theoretical model chemistry is second-order Moller–Plesset (or many-body) perturbation theory (MP2).²⁷ Since it is a fundamental property of the HF wave function that it has zero Hamiltonian matrix elements with single excitations, the leading corrections due to electron correlation are doubly substituted determinants which correspond to electron pair excitations and thus contribute to correlations between electron pairs. In MP2 theory, these correlations are described perturbatively, leading to a simple and relatively tractable expression for the correlation energy that is moderately successful for cases where the HF determinant is a qualitatively correct starting point, recovering very roughly 80% of the correlation energy per electron pair on average. Based on 100 kJ/mol per electron pair, this will generally not take us to chemical accuracy (at

least not without cancellation of errors)! Of course, the perturbation series can be carried to higher order, but it is generally preferable to seek a self-consistent treatment of electron pair correlations.

Perhaps the conceptually simplest method to self-consistently treat pair correlations is configuration interaction, CISD, where the wave function is a linear combination of the HF determinant with all determinants formed by single and double orbital substitutions, with coefficients determined variationally.²⁸ CISD cannot scale extensively with size, because the product of two fragment CISD wave functions is not itself a CISD wave function. (It contains triple and quadruple excitations.) The most effective way around this dilemma is to consider a trial wave function in which the single and double excitation operators are exponentiated, which defines the coupled cluster method,²⁹ limited to single and double replacements (CCSD).³⁰

$$\Psi_{\text{CCSD}} = \exp(T_1 + T_2)\Psi_{\text{HF}}$$

CCSD is properly extensive with molecular size, but the single and double substitution coefficients contained in the T_1 and T_2 operators are not variationally determined. As we discuss in section 5, it is even possible to define a related theory for electronic excitation energies from the ground state.

For molecules at their equilibrium geometry, CCSD recovers on the order of 95% of the ground state correlation energy with a given basis. For local chemical transformations this allows us to approach chemical accuracy. (Since absolute errors increase with the number of electrons, chemical changes involving larger numbers of electrons generally yield larger errors in relative energies also.) A further reduction of between a factor of 5 and 10 in the remaining error is possible when CCSD is corrected with a perturbative estimate of the leading contributions of the neglected excitations (the triples) termed CCSD(T).³¹ This is a sufficiently accurate treatment of electron correlation for molecules near their equilibrium geometries that chemical accuracy in relative energies is often attained, provided sufficiently large basis sets are employed.

While a DZP basis can be adequate for HF calculations, much larger basis sets are necessary to achieve the results asserted above. The larger basis set demands are a reflection of the fact that both occupied and unoccupied orbitals must be accurately described to yield the correlation energy, while for the HF method, only the occupied orbitals affected the energy. To ease the heavy computational burden of such calculations, composite schemes which evaluate the effect of basis set extension by lower levels of theory, including the MP2 method, have been introduced. The Gaussian-1 and Gaussian-2 procedures are the preeminent examples in this regard.³² These theories and other correlation techniques are discussed in more detail in a related article of this issue.⁵

The MP2, CCSD, and CCSD(T) methods form a hierarchy of size-consistent correlation treatments that introduce successively more refined (and computationally demanding) treatments of electron correlation, all beginning from a HF starting point. The primary deficiency of these methods is that while they yield chemical accuracy for the potential energy surface in the vicinity of equilibrium geometries, they cannot in general describe reaction coordinates for bond breaking to the same level of accuracy. This is because the starting point, the HF determinant, becomes suspect in bond breaking. Two separated H atoms must be described as a linear combination of *two* determinants to yield the correct energy and still be properly singlet spin adapted. Some improvement is obtained by allowing the alpha and beta orbitals to be different (unrestricted) if this lowers the energy. The dissociated products are then described in a

qualitatively correct manner, but at the price of no longer having correct spin multiplicity.

Considerable effort has therefore been devoted to the development of methods which are based not on a single determinant, like the HF method, but on the multiple determinants necessary to properly dissociate a molecule into atoms or fragments. Such methods are *multiconfigurational* self-consistent-field (MCSCF) methods, in which both orbitals and configuration interaction coefficients are determined variationally.³³ The promise of MCSCF methods is the possibility of describing a *global* potential energy surface, with reasonably uniform accuracy. By construction, our MCSCF wave function will be able to describe both equilibrium and dissociative regions of a PES. The choice of configurations can also be made such that both ground and a number of excited states can be described in a qualitatively correct manner. Hence, a second main application of MCSCF methods is the study of electronic excited states.

MCSCF calculations do not achieve high accuracy because for feasibility they involve a relatively small number of distinct configurations relative to the total number of possible configurations. After all, we are simply doing FCI if they are all included. Hence, a second step is necessary to achieve high accuracy, and that is to view the MCSCF wave function as a starting point (reference wave function) and correct it for neglected correlation effects in much the same spirit as MP2, CISD, and CCSD corrected the HF wave function. If done by configuration interaction, this is usually termed multireference CI (MRCI).³³ Large scale MRCI wave functions can now be efficiently calculated for small molecules, and this is the current method of choice for using electronic structure calculations to calculate all (or any significant part) of a PES. This is a very significant application in the context of chemical reaction dynamics, where the PES is input to a classical, semiclassical, or fully quantum dynamics procedure.³⁴

From the perspective of a theoretical model chemistry, the MCSCF and MRCI methods are problematical, because they require additional input beyond simply basis set, electrons, and nuclei. The choice of reference configurations means that they cannot be applied in an automatic fashion. This may lead to results which are not size consistent or even fail to yield a continuous PES. It is also difficult to compare calculations performed with different selection criteria. One automated approach to configuration selection which addresses these concerns is the full valence complete active space (CAS) method, which includes in the MCSCF wave function all excitations from valence occupied orbitals to valence virtual orbitals (like FCI in a minimum basis). Unfortunately, this can only be applied to small molecules because it is akin to FCI, and therefore more severely truncated choices of the CAS are commonly employed. In conjunction with a second-order perturbative correction, termed CASPT2, these methods have yielded impressive results for electronic excited states.³⁵

Achieving chemical accuracy by solving the electron correlation problem is one of the most important achievements of modern electronic structure theory. Yet clearly the story regarding the vertical axis of Figure 1 is not yet over, despite intensive development. The sequence of single reference methods HF, MP2, CCSD, and CCSD(T) cannot yield global potential energy surfaces, despite their success at equilibrium geometries. Even there, reaching chemical accuracy requires very large AO basis sets, which coupled with the high cost of the calculations means they are restricted to relatively small molecules at present (see also section 6). MCSCF methods offer advantages for the description of global potential energy surfaces and excited states, but do not meet the criteria of a theoretical

model chemistry in a feasible way. There are also promising alternative approaches such as quantum Monte Carlo methods⁵ and methods that incorporate explicit functions of the inter-electronic distances in the wave function, which are not yet mature enough to be usable by nonspecialists.

4. Density Functional Theory

The electron correlation treatments discussed in the previous section are all based on approximating the many-electron wave function. This is certainly the approach which has historically been the subject of most research effort in the molecular electronic structure theory community. However, over the past 5–10 years, an alternative approach has become increasingly successful and important in quantum chemistry. This approach, density functional theory (DFT),³⁶ has as its objective the direct determination of the exact ground state energy and electron density, without the intermediary of a many-electron wave function (although we shall see that in current form, orbitals akin to the molecular orbitals of Hartree–Fock theory still enter). Considering that the electron density is only a function of 3 variables, whereas the n -electron wave function is a function of $3n$ variables, DFT clearly has the potential to dramatically simplify electronic structure calculations. In this section I shall discuss the development and current status of DFT in highly simplified terms, noting that more detailed material can be found elsewhere in this issue.⁶

The two fundamental theorems which underpin all DFT methods were put forward by Hohenberg and Kohn some 30 years ago.³⁷ The first Hohenberg–Kohn theorem states that the exact ground state energy of a molecular system is a functional only of the electron density and the fixed positions of the nuclei. In other words, for given nuclear coordinates, the electron density uniquely determines the energy and all properties of the ground state. What this theorem does not tell us, of course, is *what* is the functional dependence of the energy on the electron density function. It merely assures us of the *existence* of the functional for the molecular ground state. Given the functional, Hohenberg and Kohn also proved that the exact electron density function is the one which minimizes the energy (i.e., as functional of the density), thereby providing a variational principle to find the density.

The challenge of DFT is the design of accurate functionals. This is a tremendously difficult problem, and yet remarkable progress has occurred over the past several decades. The total energy, deriving from the solution of the Schrödinger equation, can be decomposed in terms of kinetic energy, T , electron–nuclear attraction, V_{en} , and electron–electron interaction contributions (plus the nuclear–nuclear repulsion energy, which is constant at a given geometry). The electron–electron interactions merit additional comment. They consist of the classical Coulomb repulsion, J , due to the electron density, plus nonclassical terms due to the *correlations* between electrons and *exchange* effects of Fermion statistics. Thus, we have

$$E(\rho) = T(\rho) + V_{\text{en}}(\rho) + J(\rho) + V_{\text{xc}}(\rho)$$

Of these, V_{en} and J are straightforward, since they represent classical Coulombic interactions. T and V_{xc} are not straightforward and represent the primary problems of functional design in DFT.

A major breakthrough was provided in 1965 by Kohn and Sham, who showed that it was possible to largely bypass the difficulty associated with constructing a kinetic energy density functional by a clever reformulation of DFT.³⁸ This reformulation is termed Kohn–Sham (KS) density functional theory and

is the framework in which virtually all current density functional methods are constructed. The Hartree–Fock single determinant wave function is exact for the simple albeit unphysical case of electrons that do not interact, since there are no electron correlations. In this case, the HF expression for the kinetic energy in terms of orbitals is also exact. Kohn and Sham proved that it is possible to construct an artificial reference system of *noninteracting* electrons which has exactly the same electron density as the real molecular system of interacting electrons. The kinetic energy is approximated as that of the noninteracting reference system, which can be exactly evaluated in terms of the orbitals (termed the Kohn–Sham orbitals). The definition of the exchange–correlation (XC) functional is altered to include the kinetic energy difference between the real and noninteracting systems, which is relatively small.

Given an exchange correlation functional, a KS-DFT calculation proceeds isomorphically to a HF calculation, in that one iteratively solves for the KS orbitals which yield a self-consistent field and an associated electron density that minimizes the DFT energy. However, if the XC functional is exact, then so too is the final energy. Hence, from the point of view of a practical calculation, KS-DFT calculations are distinguished from HF calculations only by the form of exchange and correlation treatment (exact nonlocal exchange and no correlation in HF versus an exchange–correlation functional of the density in DFT, as discussed below).

At present, the design of XC functionals cannot be performed in a manner which is as rational and systematic as the development of wave function-based correlation techniques. One place to begin is the uniform noninteracting electron gas, for which it is possible to exactly obtain the exchange energy as a function of the density, a problem first solved by Dirac in 1930.³⁹ The exchange energy is proportional to $\rho^{4/3}$ in this uniform electron gas approximation. If no correlation energy functional is employed, this form of KS-DFT becomes equivalent to the earlier X- α method,⁴⁰ which had been put forward on a physical rather than mathematical basis. If a correlation functional fit to the uniform electron gas is employed, this defines the so-called local density approximation (LDA).³⁸ While the LDA is sometimes an improvement over HF theory, particularly for metallic systems including organometallic species, the improvement is not usually dramatic. The LDA in fact was employed primarily in the solid state physics community for band structures, rather than in quantum chemistry for molecular applications.

The next real breakthroughs in the development of KS-DFT are the ones responsible for its rapidly growing acceptance in the molecular electronic structure community over roughly the past decade. A new and greatly improved functional for the exchange energy was proposed in 1988,⁴¹ based on considering the exchange energies for rare gases in addition to the known behavior for the uniform electron gas. This exchange functional is commonly referred to as B-88 and depends on both density and density *gradients*. Improved functionals for the correlation energy have also emerged by considering corrections to the uniform electron gas that involve gradients of the electron density, and there is considerable activity in the development of generalized gradient approximations (GGA's) based on these ideas.⁴²

The combination of gradient-corrected correlation functionals with the Becke exchange functional has been shown to yield significantly more accurate relative energies than the earlier LDA-based forms of KS-DFT.⁴³ With these developments, DFT is now a powerful and very efficient tool for solving many problems in electronic structure theory at an accuracy roughly

comparable to the MP2 method discussed in section 3. However, because of avoiding explicit construction of the molecular wave function, the basis set requirements for DFT⁴⁴ are far more modest than those needed to obtain reliable results via the MP2 method (or any other correlation method discussed in section 3). Furthermore, for a given basis set, the computational demands of KS-DFT methods are significantly lower than conventional correlation methods. Instead, they are roughly equivalent to those of HF calculations, as discussed further in section 6. Perhaps the best DFT-based results to date have been obtained by mixing a small part of the exact exchange interactions (as modeled in the HF method) with gradient-corrected exchange functionals.⁴⁵

Overall, state-of-the-art DFT methods are at present the preferred method for treating large molecules where the far more computationally demanding electron correlation methods described in section 3 are not yet feasible. At the same time, DFT faces challenging hurdles in the future. It is by no means clear how to rationally improve the current functionals, although this is sure to be an area of considerable activity in the future. Thus, by contrast with electron correlation methods, there is no well-defined way to improve upon a given KS-DFT calculation. Furthermore, DFT at this point is an explicitly ground state theory, which has yet to be successfully extended to excited states or for that matter to time-dependent properties of the ground state (whose evolution is rigorously known in terms of wave functions or density operators, but not in terms of the electron density itself).

5. Molecular Properties

All of the discussion in the preceding sections has concerned the ways by which an electronic energy may be obtained by approximately solving the Schrödinger equation. While energy is undoubtedly the fundamental quantity, we often characterize molecules and their behavior by other properties. We might think of properties such as the dipole moment and polarizability that characterize features of the molecular charge distribution and those that characterize the structure, such as the molecular geometry and vibrational frequencies. Other molecular properties of interest might include magnetic shielding and spin–spin splittings or time-dependent quantities, such as frequency-dependent linear and nonlinear optical properties, etc. The ability to accurately and routinely calculate this wide range of molecular properties is one of the most important practical advances that has occurred in molecular electronic structure theory over the past quarter century.

The important common thread connecting the smorgasbord of molecular properties mentioned above is that they are all *responses* of the molecule to the external parameters in the Born–Oppenheimer Hamiltonian. These parameters, nuclear coordinates, applied electric and magnetic fields, etc., then become the variables on which the resulting Born–Oppenheimer electronic potential energy surface (PES) for a given theoretical model chemistry depends. Therefore, formulating analytical derivatives of the energy with respect to these variables yields a wealth of molecular properties, as summarized in Figure 2. While the energy derivative formulation is not as familiar as obtaining first-order properties as expectation values and second-order properties via second-order perturbation theory, the results are of course equivalent for exact wave functions.

As must be clear from the preceding sections, we never deal with exact wave functions in practice. Sometimes, as in DFT, we may not even have a wave function at all. Therefore, it is only via the analytical derivative formulation that we can calculate this wide range of properties in a manner which is

Energy derivative with respect to...			Defines the...		
Nuclear Position			Nuclear forces		
Electric Field			Electric dipole moment		
Electric field gradient			Quadrupole moment		
Nuclear position	&	Nuclear position	Force constants		
Electric field	&	Electric field	Electric polarizability		
Nuclear position	&	Electric field	Infrared intensities		
Magnetic field	&	Magnetic field	Magnetic susceptibility		
Magnetic field	&	Nuclear spin	Chemical shielding		
Nuclear spin	&	Nuclear spin	Spin-spin couplings		
Nuclear position	&	Nuclear position	&	Nuclear position	Anharmonic couplings
Nuclear position	&	Electric field	&	Electric field	Raman intensities
Electric field	&	Electric field	&	Electric field	First hyperpolarizability

Figure 2. Definitions of a number of chemically significant molecular properties as derivatives of the energy with respect to external parameters, such as nuclear geometry, applied fields, and nuclear spins. First-order properties are first derivatives of the energy with respect to an external parameter, second-order properties are second derivatives of the energy with respect to a pair of external parameters, and so on. As discussed in section 5, not only are these properties typically measured in the laboratory by varying external fields, but this is also how they are most conveniently calculated within a theoretical model chemistry, which, after all, provides a specification of the energy as a function of these external parameters.

fully consistent with the level of approximation used for the energy itself. For second-order properties we are freed from requiring all excited states of the unperturbed system, as is formally required in second-order perturbation expressions. Primarily for these reasons, an entire branch of electronic structure theory has grown up devoted to obtaining molecular properties as analytical energy derivatives.^{146,47} The details of this work are intricate, technical, and critical to its success, but for present purposes it suffices to understand the simple conceptual basis of analytical derivative theories.

Analytical first derivatives first become routinely available at the HF level⁴⁸ in the mid-1970s. The immediate implication was that instead of studying potential energy surfaces point by point, it became possible to readily *minimize* the energy of a molecule with respect to nuclear coordinates and study molecular structure within different theoretical model chemistries. The efficient and accurate first principles prediction of molecular structure is a key accomplishment of electronic structure theory. Even at a relatively simple level of theory, the results are impressively accurate. The uncertainties in calculated bond lengths and angles are typically 0.01 Å and 1° at the HF/6-31G* (6-31G* is a double zeta basis augmented with polarization functions) level of theory for organic molecules.²⁰ This accuracy increases at the MP2 level and improves still further at the CCSD(T) level, where molecular geometries are usually almost as accurate as it is meaningful to examine.

With the availability of analytical second derivatives of the HF energy⁴⁹ in the early 1980s, it then became possible to study the curvature of potential energy surfaces at stationary points and thereby characterize them as either minima (all eigenvalues of the second-derivative matrix are positive) or *k*th-order saddle points (*k* negative eigenvalues). Second derivatives greatly facilitate the location of transition structures (*k* = 1), with obvious implications for the study of reaction mechanisms. Furthermore, via normal mode analysis, calculated force constants yield harmonic vibrational frequencies that may be directly

compared with experimental infrared and Raman spectroscopy. The HF method is useful for vibrational frequencies, although it typically overestimates bond stretch vibrations by roughly 10%, a trend that is sometimes corrected by scaling. KS-DFT with today's best functionals is a significant improvement over HF calculations, as is MP2 theory. For molecules where CCSD(T) calculations are possible with large basis sets, sufficient accuracy is obtained to usually permit unambiguous assignment of experimental infrared and Raman transitions.⁵⁰ A number of misassignments of such transitions have been corrected by accurate ab initio calculations!

Similar progress is occurring in the use of electronic structure methods to calculate the other properties delineated in Figure 2. It is beyond my scope to review them in any detail at all here, other than to note the exciting progress in calculating NMR properties⁵¹ and electrical properties such as dipole moments and electrostatic potentials. Induced electrical properties such as static and frequency-dependent polarizabilities seem to pose greater challenges for electronic structure calculations,⁵² and to date ab initio quantum chemical methods have not played a significant role in associated areas of chemistry such as the design of novel molecules for nonlinear optical applications. This is an area where we may expect progress in the future.

There are beautiful connections between response theory for static response properties, frequency-dependent properties, and excitation energies because the same response matrices govern all three classes of problem.^{12,53} In the case of excitation energies, this gives us a simple and elegant method of generalizing a successful ground state wave function approximation to excited states. Imagine applying a time-dependent perturbation of adjustable frequency ω to a molecular system described by a ground state model chemistry and studying the first-order response of the molecular wave function as a function of applied frequency. At some particular frequencies, the response of the wave function will become very large, and diverge, and we can immediately identify such frequencies as being the Bohr frequencies of the molecule within the theoretical model chemistry! This is the physical basis for generalizations of the HF and CCSD methods to excitation energy calculations and is another area where significant progress is occurring. It permits us to imagine an analog of Figure 1 for calculations of electronic excited states.

6. Bottlenecks to Treating Large Molecules

Someone looking at the electronic structure field from outside might rightly speculate that the widespread adoption of electronic structure methods is in large measure a result of the dramatic progress that has occurred in the availability of very high speed computing resources. Today's high performance workstations are almost 3 orders of magnitude faster than the minicomputers and early workstations of 10–15 years ago. In the realm of supercomputing, we are just now witnessing the emergence of usable massively parallel computers which is an exciting development with strong implications for the future of this field.⁵⁴ The future rate of improvement in computing (performance is currently doubling roughly every 2 years) might well appear to be the primary determinant of how the impact of electronic structure calculations will grow in the future.

If this is not entirely the case, then what else is important? In a related vein, is it or is it not feasible to think of ab initio calculations on molecules the size of proteins by early in the next century? In this part of the article we scratch beneath the surface of the theoretical model chemistries enough to try to answer these questions and, in the process of doing so, reveal some of the remarkable progress that has occurred in the

Theoretical method	Current computational dependence on molecular size, M	Current estimate of maximum feasible molecular size
FCI	factorial	2 atoms
CCSD(T)	M^7	8-12 atoms
CCSD	M^6	10-15 atoms
MP2	M^5	25-50 atoms
HF, KS-DFT	M^2 - M^3	50-200 atoms

Figure 3. Current scalings of electronic structure theory methods with molecular size, M , and estimates of the maximum molecular sizes (in terms of numbers of first-row non-hydrogen atoms) for which energy and gradient evaluations can be tackled by each method at present. The latter estimates are *very* approximate because they depend critically on many variables beyond simply the size dependence of computational complexity and the assumed availability of a high-end workstation. Other factors include the number of energy and gradient evaluations required (assumed to be fewer than 10), the size of the basis set chosen (assumed to be DZP quality), molecular symmetry (none assumed), etc. Typical current calculations are of course on *substantially smaller* molecules at each level of theory. Nevertheless, the overall pattern of steep reduction in maximum feasible molecular size with increasingly sophisticated electron correlation treatments is an accurate reflection of the current limits of electronic structure methods. It also illustrates the need for new algorithms which scale more physically with molecular size, as discussed in section 6.

algorithms for much of electronic structure theory. We shall also see, however, that in many areas equally remarkable progress is still required in the future! As a rough guide to the present state of affairs, Figure 3 contains a summary of the current scalings with molecular size of some of the theoretical models discussed in sections 2–4 and estimates of the maximum size of calculation feasible, subject to various assumptions.

If we consider the series of theoretical models, HF (or KS-DFT methods), MP2, CCSD, and CCSD(T), for a given size basis set and varying molecular size, M , then in the simplest analysis their computational requirements scale as M^4 , M^5 , M^6 , and M^7 , respectively. For example, M^5 scaling implies that doubling the size of the molecule leads to a calculation roughly 32 times longer. Let me discuss the changing views of computational bottlenecks in HF calculations (KS-DFT calculations may be viewed essentially identically) as a first example to show some of what has been achieved and is being achieved. Two basic steps are involved, which are repeated iteratively until the self-consistent field is achieved and the solutions no longer change. The first step is the construction of the effective one-electron Hamiltonian matrix (usually termed the Fock matrix), given a density matrix. The second is the generation of a new density matrix, usually via the generation of new molecular orbitals or Kohn–Sham orbitals. How has our ability to evaluate these two steps improved?

The assertion that the HF method scales as the fourth power of molecular size arises because of the evaluation of electron–electron interactions via four-center, two-electron integrals. The density consists of sums of *products* of AO's, and electron–electron interactions are thereby the sum of a quartic number of Coulomb interactions between *pairs of pairs* of AO's. For Gaussian AO's these two-electron integrals can be analytically evaluated, and there is a large literature on the successful efforts to reduce their computational cost.⁵⁵ More important, however, is the fact that the number of *nonnegligible* two-electron

integrals does not grow quartically with the size of the molecule. Rather, it grows quadratically (M^2), when the molecular size is large enough, because the two AO's comprising each pair must overlap in order to make a distribution containing nonnegligible charge. This realization together with advances in the speed of two-electron integral evaluation combine to permit routine calculations on systems approaching the 100 atom range. Calculations of this size have become possible through the direct SCF method⁵⁶ in which integrals are generated as they are needed rather than stored, which exploits the fact that computational power is increasing far more rapidly than input/output capability.

The generation of a new density matrix via diagonalization scales as M^3 , but with such a small coefficient that two-electron integral evaluation still dominates practical molecular calculations. Very recently, this has begun to change with the advent of new methods, such as the continuous fast multipole method,⁵⁷ for collectivizing distant electron-electron interactions via multipole expansions such that the effective Hamiltonian can be constructed to high accuracy with computational work scaling only *linearly* with M . The physical basis for achieving linear scaling in the assembly of the Fock matrix follows directly from the localized nature of AO basis functions. In the near future methods based on these and related ideas may well be routinely used to permit very large scale calculations.

On the second step of an SCF iteration, progress has been recently reported in the solid state physics literature in the form of novel methods for updating the density⁵⁸ and/or orbitals⁵⁹ without explicit diagonalization. The physical basis for approaching linear scaling in the density matrix update procedure is the fact that, for extended systems with a band gap, the density matrix is spatially localized. The extent of localization is related to the band gap, such that in the extreme case of a metal the density matrix is substantially delocalized. At least for insulators, which describes most molecules, it therefore appears plausible that HF and KS-DFT calculations will approach linear scaling in the relatively near future. We discuss the practical implications of these exciting developments at the end of this section.

I shall now consider the MP2 method, the simplest wave function-based theory of electron correlation, as the next step upward in complexity. In most current quantum chemistry program packages MP2 scales as M^5 . This is an unphysical scaling for most systems where electron correlation, just as one might expect, is primarily a local property that results from electrons avoiding each other at close range. The success of KS-DFT where the XC functional describing electron exchange and correlation effects is spatially local is essentially a corollary of this assertion. The M^5 scaling is a consequence of the *delocalized* MO's which arise from standard HF calculations.

Any attempt to reformulate MP2 theory to scale in a more physical manner with molecular size must overcome the problem of delocalized MO's. Fortunately, the MO's can be localized, as is discussed in more detail in section 8, and there has been some preliminary but very promising progress toward developing versions of MP2 theory based on localized orbitals. The "local-MP2" method⁶⁰ scales only quadratically with molecular size and comes to within a few percent of reproducing the exact MP2 energy with a given basis. Much effort will be dedicated in the near future to reformulating MP2 and the higher level standard electron correlation methods so that their computational complexity increases if possible only linearly, and certainly not worse than quadratically with molecular size for large molecules. On physical grounds at least, the outlook for success appears very promising.

Let us now ask whether an ab initio electronic structure calculation on a protein might be possible early in the next century. At present, a HF or DFT calculation on a system of 100 atoms is feasible. I consider a protein of 10 000 atoms as the target size, a factor of roughly 100 times larger, and assume a 100-fold increase in computing resources. Therefore, only under the presumption that we have true linear scaling of the computational complexity with molecular size will such a calculation appear even remotely feasible.

A related question is what difference would a 100-fold improvement in computing resources make in the size of molecule which we can study by a relatively high level correlated method such as CCSD, given the way it presently scales with molecular size? This scaling is strictly a sixth-power law at present, so the increase in molecular size will be the sixth root of 100, or just over 2. Clearly without dramatic improvements in the underlying algorithms for performing such calculations, we cannot expect very large changes in the size of molecular systems which can be treated at this level of theory.

7. The Effect of Environment

To this point, we have viewed the deficiencies of electronic structure theory in terms of the two central approximations introduced in section 2, namely, the correlation treatment, discussed there and in sections 3 and 4, and the quality (or extent of completeness) of the basis set used to represent the molecular orbitals. This is not strictly valid for various reasons. For example, we have implicitly neglected the role of relativistic effects, which can be a significant source of error for very heavy elements. We have also assumed validity of the Born-Oppenheimer approximation and largely neglected the role of nuclear motion, which is unsatisfactory in some situations. Nevertheless, for calculating and analyzing the structure, relative energies, and other properties of isolated molecules (or even molecules that interact only weakly with their environment), this is generally an adequate procedure.

How is the situation where a molecule interacts strongly with its environment dealt with in electronic structure theory? Clearly, the treatment of the environment must clearly be considered a third approximation of at least equal significance to the two we have considered hitherto. Let me pose some physical examples which can be used to measure the progress that has been made and the extent to which this is an area ripe for further work. We might first think about cases where a molecule of interest is relatively weakly affected by an environment. Examples might include a molecule that has adsorbed from the gas phase onto a surface without dissociating or a solvated molecule in a relatively nonpolar solvent such as octane. More difficult cases are those where the molecule is so strongly interacting with its environment that it is no longer a distinct entity. Examples might include the problem of a reaction center in a large protein, such as chlorophyll embedded in the cellular matrix, a defect in a disordered solid, or even types of solvent-assisted reactions in solution or surface-assisted catalytic processes.

One can distinguish two main levels at which the environment may be treated, which we discuss in turn in the context of the types of examples listed above.

7.1. Environment Modeling at the Same Level as the Molecule Is Treated. While this must be the ultimate objective, it is at present virtually never done with a full model of an extended environment, as it is unfeasible.⁶¹ The ab initio molecular dynamics methods⁶² discussed elsewhere in this issue⁶³ are now just becoming possible for simulations of small, periodically replicated models of liquids. The state of the art

in this area is illustrated by a recent simulation⁶⁴ of the dissociation of an HCl molecule in a box containing 32 water molecules, which was simulated by propagating Newton's equations of motion for the nuclei for over 10 000 time steps (roughly 2 ps of real time), solving the electronic structure problem at each time step. While size of system and time scale of simulation are not yet sufficient to calculate reliable thermodynamic properties, they can already give valuable insight into the short-time dynamics of the system. Furthermore, substantial progress in this "first principles" level of modeling of solvation phenomena will occur in the near future.

Tremendous simplifications potentially arise in the problem where the environment is ordered rather than fluctuating, as in the model problem of a reaction occurring on an ordered solid surface. Band structure techniques can be used to efficiently solve for the electronic structure of the extended surface in the absence of the adsorbed species.⁶⁵ The problem of molecule-surface interaction then becomes one of "embedding" a defect region in which the chemistry of interest occurs into the undisturbed electronic structure of the surrounding bulk. While there has been considerable theoretical activity on this problem,⁶⁶ it has not yet been solved in a way which permits routine application to problems in surface chemistry, and at present most effort is directed toward the perhaps simpler alternative approach of making calculations with larger unit cells feasible.

So much for modeling the environment in a manner which involves no additional approximation. At present, the most common way of treating the electronic structure of the environment at the same level as that of the molecule of interest is to make the calculation feasible by drastically truncating the model of the environment. This gives rise to so-called "cluster models" of the surface,⁶⁷ where between 1 and perhaps 10–100 "surface atoms" interact with the molecule, or to solvent models in which only a shell or several shells of solvent molecules are explicitly treated.

Cluster modeling of the environment is a little like configuration selection in the MCSCF method we discussed earlier in section 3. In the hands of experts with a good feel for the nature of the molecule-environment interactions, good results can be obtained, but it is relatively easy to obtain ambiguous or misleading results. It is hard to assess the physical significance of the results except by comparing observables with experiment. Therefore, truncated cluster models of either surfaces or solvent environment are often best used for explanatory rather than predictive purposes. As an example, considerable insight into the bonding of simple diatomic molecules to surfaces has been obtained from cluster calculations, using clusters which were shown to reproduce key experimental observables such as chemisorption bond energies, binding sites, and associated vibrational frequencies.⁵⁷

7.2. Environment Modeling at a Simplified Level. If the molecule of interest is interacting only relatively weakly with its surroundings, as in the case of solvation in nonpolar solvents or physisorption of molecules on surfaces, then it is not unreasonable to model the environment at a (sometimes greatly) simplified level. Simplified models of the environment are more problematical when there is a strong chemical interaction between the molecule and its environment or when a molecule and local environment are bonded to the more distant environment.

The case of weak interaction is of significance for solvation modeling. Much effort has been devoted to simplified treatments of the solvent which retain primarily the electrostatic interactions with the solute. Classically, the bulk solvent may be characterized as a polarizable medium with a dielectric

constant whose magnitude is connected to the polarity of the solvent. "Continuum" solvation models⁶⁸ (also known as reaction field methods) treat the problem of a molecule whose electronic structure is described by a theoretical model of the type discussed earlier, embedded in a dielectric medium which interacts electrostatically with the molecule to alter the electron distribution from its gas phase form. The solute is typically enclosed in a cavity which might be specified in any of a myriad of ways, for example, as a set of interlocking atom-centered spheres whose size is given by van der Waals radii.

As an example of the success of these models, consider the problem of tautomeric equilibria. Qualitatively correct solvent effects have been obtained even for problems as demanding as the relative energies of neutral and zwitterionic forms. However, the reader should be aware that all results obtained with continuum solvation models are somewhat dependent on the shape of the cavity in which the molecule is embedded in the dielectric solvent and that the specification of the cavity often involves empirical parameters. Furthermore, quantum mechanical interactions including dispersion forces, exchange interactions, and so forth between molecule and solvent are omitted (or modeled in an empirical fashion), as are potentially important effects due to solvent dynamics.

The next step for modeling the environment is to retain the atomic detail but merely treat the electronic structure at a much reduced level.⁶⁹ The minimal level of atomic modeling of the environment would be to employ an empirical potential energy function, such as are readily available for organic and biological molecules (often called molecular mechanics potentials), coupled to the electronic structure problem for the molecule or reaction center. For example, one simple procedure for combining an electronic structure calculation on a central "reaction center" with a molecular mechanics treatment of the extended environment involves defining a composite energy in the following way. It is the sum of the electronic structure energy for a calculation on the reaction center (with bonds connecting this region to the environment replaced by appropriate numbers of terminating hydrogen atoms) plus the energy calculated by the empirical force field, excluding contributions associated with the reaction center, which would otherwise be double counted.⁷⁰ The composite energy function can be minimized with respect to geometric parameters or alternatively used in molecular dynamics or Monte Carlo simulations.

There is much scope for imaginative future developments in this area. Specifically, the environment might be modeled by a relatively low level of electronic structure theory such as self-consistent-field theory with a basis set of modest size, while the "system" is treated at a much higher level of electronic structure theory, perhaps including details of electron correlation. The critical issue in all such modeling is how to couple the two regions together in as consistent a fashion as possible. In this regard the ongoing developments in electronic structure algorithms discussed in the previous section, which are focused on localized quantities, are significant. Describing the equations of self-consistent-field theory or electron correlation in terms of localized quantities lends itself far more naturally to only treating a target region by such a method than is the case when orbitals are fully delocalized.

8. Chemical Insight from Electronic Structure Calculations

The primary purpose of *any* electronic structure calculation is insight of one sort or another. Often the insight relates directly to the fact that an electronic structure calculation is itself a numerical experiment, which can reliably calculate observable

quantities, which can subsequently be interpreted in chemical terms. For example, locating transition structures for two different dissociation pathways in a gas phase reaction will explain why one is preferred to the other in terms of the barrier height and the arrangement of atoms at the competing transition structures. Alternatively, molecular properties such as dipole moments or chemical shielding, etc., which can be calculated via electronic structure methods can indirectly characterize chemical bonding. Cases like these show the value of being able to apply reliable electronic structure methods to yield observables, like the transition structures, that are not directly accessible by other, experimental, means.

Since the results of high quality electronic structure calculations give either wave functions or densities that can approach the exact quantities, it is also interesting to ask what additional insight into the bonding can be obtained from all this information. To what extent is it possible to learn about the bonding from the wave function, which is not directly observable, as well as the density, which in principle is observable? Have new paradigms for thinking about bonding emerged from this field, or is it becoming primarily the development of numerical machinery? This question of the chemical interpretation of wave functions and electron densities has stimulated considerable work over the past several decades.

At the most fundamental level, the question of the origin of the chemical bond has been given an energetic interpretation, in terms of the changes in kinetic energy and potential energy as two nuclei approach each other.⁷¹ A very brief summary of the main results of this type of energy component analysis is as follows. The electron sharing in a covalent bond arises in the first instance because of reduced kinetic energy due to the expanded volume of the orbital. (This is akin to the dependence of the particle in a box kinetic energy on box length.) This permits the electron to be drawn closer to the nuclei than is possible in the atoms and overcomes the potential energy increase associated with electron sharing. This type of analysis has been performed for specific cases but is not currently employed as a general tool for discussing trends in bonding.

In terms of discussing chemical bonding in terms of wave functions, one must face the fact that exact wave functions, as discussed in section 2, are functions of the $3n$ positional coordinates of the n electrons. We cannot visualize functions in more than three dimensions, and therefore the first necessity is to consider functions of at most three variables at once, like the molecular orbitals of HF theory or the Kohn–Sham (KS) orbitals of density functional theory. One drawback of looking at either HF or KS orbitals is that they are not unique. Taking the sum and the difference of two occupied MO's gives two equally acceptable occupied MO's (after renormalizing), for example. This nonuniqueness is clearly problematic for using orbitals as a metaphor for interpreting chemical bonding.

Different criteria are available to fix the form of the MO's. As one might expect, some are appropriate for one problem and others for another, often by design. The most familiar form of the orbitals are the "canonical orbitals" which diagonalize the SCF Hamiltonian and are the end results of current HF and KS-DFT calculations. For the ionization of an electron from a molecule, the HF canonical orbitals are appropriate, because via Koopmans' theorem,⁷² ionization potentials correspond approximately to the energy difference between the original HF determinant and determinants formed by vacating canonical orbitals. Canonical orbitals corresponding to interacting fragments are also employed in so-called "perturbational molecular orbital" analysis.⁷³ However, the canonical orbitals are delocalized, regardless of the size of the molecule, which is not in

accord with the Lewis-derived picture of localized electron pair bonds and lone pair orbitals.

Therefore, various localization criteria^{74,75} have been put forth to yield localized orbitals which might be more appropriate for thinking about chemical bonding (and may also help in attacking the open issues discussed in section 6). It is beyond my scope to review them here, but it is worth noting that quite different localization criteria often yield quite similar localized orbitals and that in organic molecules, for example, the localized valence orbitals generally correspond to bond orbitals or lone pairs, in general accord with traditional Lewis electron pair pictures of bonding. It is also possible to study the Kohn–Sham orbitals of KS-DFT methods in the same way, although they refer strictly to an unphysical reference system of noninteracting electrons! In the electron correlation techniques discussed in section 3, the wave functions consist of many determinants, and therefore the value of studying orbitals is reduced still further as we no longer have strictly occupied and strictly unoccupied orbitals, but rather fractional occupancies. The fractionally occupied orbitals are termed "natural orbitals".⁷⁶

To circumvent difficulties of wave function analysis, much effort has been devoted to studying properties of chemical bonding beginning *directly* from the electron density, which is an observable function of only three variables. To simply characterize the molecular charge distribution and the nature of bonding, it is worthwhile to attempt to partition the density into atomic charges (which are of course not uniquely defined and not observable). Various methods for doing this are available, and while there is inevitably controversy over the manner in which a nonobservable quantity should be (artificially) defined, these schemes are of considerable interpretive value when employed cautiously. Perhaps the most elegant scheme for partitioning electron density between atoms is a topological analysis⁷⁷ of the density which defines interatomic boundaries as the curve where the gradient of the density normal to the boundary vanishes. This partitioning leads to definitions of atomic properties that go far beyond simply the integrated charge.

There appear to be good prospects in the future for narrowing the gap between the methods of electronic structure theory and the tools and language for interpreting the wave functions and/or densities that emerge from such calculations. At present, the methods are heavily based on delocalized orbitals, as we have already discussed. Yet for most molecular systems, these delocalized orbitals sum together to yield a spatially localized electron density matrix, and most interpretation of bonding is performed within localized pictures. Localized electronic structure methods may therefore connect much more directly to subsequent discussion of bonding in chemical terms.

9. Electronic Structure Theory: Present and Future

The accomplishments of electronic structure theory over the past several decades have brought these techniques very much into the mainstream of chemistry. For the calculation of molecular properties such as geometry, vibrational frequencies, charge distributions, and much more, standardized theoretical models such as Hartree–Fock theory, density functional theories, second-order Moller–Plesset perturbation theory, and coupled cluster theory form a reasonably well-calibrated hierarchy of increasing accuracy. It is possible to choose a level of accuracy which is adequate for most qualitative and many quantitative purposes, for relatively small molecules. These methods work best for molecules near their equilibrium geometries but are often useful for characterizing transition states

and higher-order saddle points on potential energy surfaces also. Multireference methods are capable of treating bond-breaking problems.

This experience coupled with the availability of standardized basis sets means that routine application of these methods to a wide range of chemical problems is now possible for nonexperts. With the proliferation of low cost workstations whose capability nevertheless greatly exceeds minicomputers of the early 1980s, electronic structure theory is in the fortunate position of having a continually decreasing cost of entry! For readers who are newcomers to this field, the good news is that it is easy to learn more and to experiment with the capabilities (and inevitably also the limitations!) of the standard theoretical model chemistries. The textbooks and reviews already cited (e.g., refs 11, 20, and 22) are a good place to begin, together with one of the many program packages (and their tutorials!) which are available, both commercially and in the public domain.

As I have stressed at various points throughout the article, this is nevertheless not a mature field in the sense of applied linear algebra, where standard algorithms are well established. Instead, almost all fundamental aspects of quantum chemistry remain to varying degrees in a state of rapid development: this includes the theoretical models, the algorithms used to implement them, the methods used to interpret them, and of course by implication the problems to which they may be applied. The future therefore appears full of challenges that when met will transform the horizons of the field. Here is a summary of some of the present issues that have been mentioned:

(1) There are no *completely satisfactory* theoretical models which are *feasible*. The most widely used methods for treating molecules near their equilibrium geometries are not adequate for global exploration of potential energy surfaces, due to being unable to correctly describe bond breaking. Methods that can correctly describe bond breaking are not formulated in a way which meets the criteria of a theoretical model chemistry without prohibitive cost. There is hence much scope for novel future developments in the area of electron correlation, with this remark applying even more strongly to excited states than for the ground state.

(2) Most widely used theoretical model chemistries have computational requirements which scale in an *unphysical* way with the size of the molecule (to be specific, worse than the quadratic scaling implied by Coulomb's law for distant electron-electron interactions). While applications of self-consistent-field methods to systems in the hundreds of atom region are now possible, and will surely become routine in the near future, much work is required to reformulate existing theories of electron correlation to permit their application to the large systems where much of the future of chemistry lies.

(3) Electronic structure methods have primarily been successful in treating *gas phase* problems or problems in which only a very local chemical environment is important. To study complex many-body problems in biological chemistry, surface chemistry, or even simply solution chemistry, much further work is needed on soundly based methods that embed a reaction center in an environment. Progress in this area will allow electronic structure methods to be more profitably exploited in the context of statistical mechanical problems and both classical and quantum mechanical time-dependent problems.

(4) While the advances in electronic structure methods have revolutionized our ability to predict molecular properties from first principles in a way which is often of adequate reliability to make chemical predictions, the resulting methods have diverged from the tools and language for interpreting the results of the calculations. There remain real challenges in extracting

insight and not merely high quality numbers from electronic structure calculations, and one unifying theme for the future may be connections between emerging "local" algorithms and localized pictures of chemical bonding and functional group properties.

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References and Notes

- (1) A remark attributed to P. A. M. Dirac in 1929, as quoted for example in: La Paglia, S. R. *Introductory Quantum Chemistry*; Harper and Row: New York, 1971.
- (2) Lewis, G. N. *J. Chem. Phys.* **1933**, *1*, 17.
- (3) Goddard, W. A. *Science (Washington, D.C.)* **1985**, *227*, 917.
- (4) Schaefer, H. F. III *Science (Washington, D.C.)* **1986**, *231*, 1100.
- (5) Raghavachari, K.; Anderson, J. B. *J. Phys. Chem.* **1996**, *100*, 12960.
- (6) Kohn, W.; et al. *J. Phys. Chem.* **1996**, *100*, 12974.
- (7) The following textbooks provide a general introduction to quantum mechanics for chemists: (a) Lowe, J. P. *Quantum Chemistry*; Academic: Boston, MA, 1993. (b) McQuarrie, D. A. *Quantum Chemistry*; University Science Books: Mill Valley, CA, 1983.
- (8) Born, M.; Oppenheimer, J. R. *Ann. Phys.* **1927**, *79*, 361.
- (9) Hartree, D. R. *Proc. Cambridge Philos. Soc.* **1928**, *24*, 328.
- (10) Fock, V. A. *Z. Phys.* **1930**, *15*, 126.
- (11) For a good textbook introduction to the Hartree-Fock method and some electron correlation techniques, see: Szabo, A.; Ostlund, N. S. *Modern Quantum Chemistry: Introduction to Advanced Electronic Structure Theory*; McGraw-Hill: New York, 1989.
- (12) For a graduate level treatment of most areas of electronic structure theory, see, for example: McWeeny, R. *Methods of Molecular Quantum Mechanics*; Academic: London, 1989.
- (13) For a historical account of the development of molecular electronic structure theory in terms of key publications, see: Schaefer, H. F. III *Quantum Chemistry: The Development of Ab Initio Methods in Molecular Electronic Structure Theory*; Clarendon Press: Oxford, 1984.
- (14) Roothaan, C. C. J. *Rev. Mod. Phys.* **1951**, *23*, 69.
- (15) Davidson, E. R.; Feller, D. *Chem. Rev.* **1986**, *86*, 681
- (16) Shavitt, I. *Isr. J. Chem.* **1993**, *33*, 357.
- (17) Boys, S. F. *Proc. R. Soc. London* **1950**, *A200*, 542.
- (18) Pople, J. A. *J. Chem. Phys.* **1965**, *43*, S229.
- (19) Pople, J. A. In *Energy, Structure and Reactivity: Proceedings of the 1972 Boulder Summer Research Conference on Theoretical Chemistry*; Smith, D. W., McRae, W. B., Eds.; Wiley: New York, 1973; p 51.
- (20) Hehre, W. J.; Radom, L.; Schleyer, P. v. R.; Pople, J. A. *Ab Initio Molecular Orbital Theory*; Wiley: New York, 1986.
- (21) Bartlett, R. J.; Purvis, G. B. *Int. J. Quantum Chem., Quantum Chem. Symp.* **1978**, *14*, 561.
- (22) Simons, J. *J. Phys. Chem.* **1991**, *95*, 1017.
- (23) A current technical treatment of many advanced topics in electronic structure theory is given in the following compilation: *Modern Electronic Structure Theory*; Yarkony, D. R., Ed.; World Scientific: Singapore, 1995; Parts I and II.
- (24) A collection of papers which is oriented to the performance of quantum chemical methods is: *Quantum Mechanical Electronic Structure Calculations with Chemical Accuracy*; Langhoff, S. R., Ed.; Kluwer: Dordrecht, 1994.
- (25) Chakravorty, S. J.; Gwaltney, S. R.; Davidson, E. R.; Parpia, F. A.; Froese-Fischer, C. *Phys. Rev.* **1993**, *47*, A3649.
- (26) See for example: Bauschlicher, C. W.; Langhoff, S. R.; Taylor, P. R. *Adv. Chem. Phys.* **1990**, *77*, 103.
- (27) Møller, C.; Plesset, M. S. *Phys. Rev.* **1934**, *46*, 618.
- (28) For an overview, see for example: Shavitt, I. In *Modern Theoretical Chemistry*; Schaefer, H. F., Ed.; Plenum: New York, 1977; kVol. 4, p 189.
- (29) Cizek, J. *J. Chem. Phys.* **1966**, *45*, 4256. Cizek, J. *Adv. Chem. Phys.* **1969**, *14*, 35.
- (30) Bartlett, R. J. *J. Phys. Chem.* **1989**, *93*, 1697.
- (31) Raghavachari, K.; Trucks, G. W.; Pople, J. A.; Head-Gordon, M. *Chem. Phys. Lett.* **1989**, *157*, 479.
- (32) Curtiss, L. A.; Raghavachari, K.; Trucks, G. W.; Pople, J. A. *J. Chem. Phys.* **1991**, *94*, 7221.
- (33) See for example articles by: Roos, B. O.; Bruna, P.; Peyserimhoff, S. D.; Shepard, R. In *Ab Initio Quantum Chemistry II*; Lawley, K. P., Ed.; *Adv. Chem. Phys.* **1987**, *67*.

- (34) Schinke, R. *Photodissociation Dynamics: Spectroscopy and Fragmentation of Small Polyatomic Molecules*; Cambridge University Press: Cambridge, 1993.
- (35) Roos, B. O.; Fulscher, M. P.; Malmqvist, P. A.; Merchan, M.; Serrano-Andres, L. In *Quantum Mechanical Electronic Structure Calculations with Chemical Accuracy*; Langhoff, S. R., Ed.; Kluwer: Dordrecht, 1994.
- (36) Parr, R. G.; Yang, W. *Density Functional Theory of Atoms and Molecules*; Oxford: New York, 1989.
- (37) Hohenberg, P.; Kohn, W. *Phys. Rev.* **1964**, *136*, B864.
- (38) Kohn, W.; Sham, L. J. *Phys. Rev.* **1965**, *140*, A1133.
- (39) Dirac, P. A. M. *Proc. Cambridge Philos. Soc.* **1930**, *26*, 376.
- (40) Slater, J. C. *Phys. Rev.* **1951**, *81*, 385.
- (41) Becke, A. D. *Phys. Rev.* **1988**, *38*, A3098.
- (42) Perdew, J. P.; Chevary, J. A.; Vosko, S. H.; Jackson, K. A.; Pederson, M. R.; Singh, D. J.; Fiolhais, C. *Phys. Rev.* **1992**, *B46*, 6671.
- (43) Becke, A. D. *J. Chem. Phys.* **1992**, *97*, 9173.
- (44) Johnson, B. G.; Gill, P. M. W.; Pople, J. A. *J. Chem. Phys.* **1993**, *98*, 5612.
- (45) Becke, A. D. *J. Chem. Phys.* **1993**, *98*, 5648.
- (46) Schaefer, H. F. III; Yamaguchi, Y. *J. Mol. Struct. (THEOCHEM)* **1986**, *135*, 369.
- (47) Yamaguchi, Y.; Osamura, Y.; Goddard, J. D.; Schaefer, H. F. III *A New Dimension to Quantum Chemistry: Analytical Derivative Methods in Ab Initio Molecular Electronic Structure Theory*; Oxford: New York, 1994.
- (48) Pulay, P. In *Applications of Electronic Structure Theory*; Schaefer, H. F., Ed.; Plenum: New York, 1977; p 153.
- (49) Pople, J. A.; Raghavachari, K.; Schlegel, H. B.; Binkley, J. S. *Int. J. Quantum Chem., Quantum Chem. Symp.* **1979**, *13*, 225.
- (50) Lee, T. J.; Scuseria, G. E. In *Quantum Mechanical Electronic Structure Calculations with Chemical Accuracy*; Langhoff, S. R., Ed.; Kluwer: Dordrecht, 1994.
- (51) Gauss, J. *Ber. Bunsen-Ges. Phys. Chem.* **1995**, *99*, 1001.
- (52) Sekino, H.; Bartlett, R. J. *J. Chem. Phys.* **1991**, *94*, 3665; **1993**, *98*, 3022.
- (53) Olsen, J.; Jørgensen, P. In *Modern Electronic Structure Theory*; Yarkony, D. R., Ed.; World Scientific: Singapore, 1995; Part II, p 857.
- (54) Harrison, R. J.; Shepard, R. *Annu. Rev. Phys. Chem.* **1994**, *45*, 623.
- (55) See for example: Gill, P. M. W. *Adv. Quantum Chem.* **1994**, *25*, 141.
- (56) Almlöf, J.; Faegri, K., Jr.; Korsell, K. *J. Comput. Chem.* **1982**, *3*, 385.
- (57) White, C. A.; Johnson, B. G.; Gill, P. M. W.; Head-Gordon, M. *Chem. Phys. Lett.* **1994**, *230*, 8; **1996**, *253*, 268.
- (58) Li, X.-P.; Nunes, R. W.; Vanderbilt, D. *Phys. Rev.* **1993**, *B47*, 10891.
- (59) Galli, M.; Parrinello, M. *Phys. Rev. Lett.* **1992**, *69*, 3547.
- (60) Saebø, S.; Pulay, P. *Annu. Rev. Phys. Chem.* **1993**, *44*, 213.
- (61) There is of course one significant exception to this statement, which is the use of ab initio calculations to parametrize empirical potential energy surfaces, either in combination with experimental data or by itself, which defines "ab initio-derived potential energy surfaces". See for example: Weakliem, P. C.; Carter, E. A. *J. Chem. Phys.* **1993**, *98*, 737.
- (62) Car, R.; Parrinello, M. *Phys. Rev. Lett.* **1985**, *55*, 2471.
- (63) Madden, P. *J. Phys. Chem.* **1996**, *100*, xxx.
- (64) Laasonen, K.; Klein, M. L. *J. Am. Chem. Soc.* **1994**, *116*, 11620.
- (65) Pisani, C.; Dovesi, R.; Roetti, C. *Hartree-Fock Ab Initio Treatment of Crystalline Systems*; Springer-Verlag: Berlin, 1988; Vol. 48 in the series *Lecture Notes in Chemistry*.
- (66) Pisani, C.; Cora, F.; Nada, R.; Orlando, R. *Comput. Phys. Commun.* **1994**, *82*, 139 (and the following articles).
- (67) Bagus, P. S.; Pacchioni, R., Eds. *Cluster Approach to the Chemistry and Physics of Surfaces and Interfaces*; special issue of *J. Electron Spectrosc. Relat. Phenom.* **1994**, *69*.
- (68) Tomasi, J.; Persico, M. *Chem. Rev.* **1994**, *94*, 2027.
- (69) An early example of such modeling is: Singh, U. C.; Kollman, P. A. *J. Comput. Chem.* **1986**, *7*, 718.
- (70) Maseras, F.; Morokuma, K. *J. Comput. Chem.* **1995**, *16*, 1170.
- (71) (a) Ruedenberg, K. *Rev. Mod. Phys.* **1962**, *34*, 326. (b) Feinberg, M. J.; Ruedenberg, K.; Mehler, E. L. *Adv. Quantum Chem.* **1970**, *5*, 28.
- (72) Koopmans, T. *Physica* **1933**, *1*, 104.
- (73) Jørgensen, W. L.; Salem, L. *The Organic Chemist's Book of Orbitals*; Academic: New York, 1973.
- (74) Edmiston, C.; Ruedenberg, K. *Rev. Mod. Phys.* **1963**, *35*, 457.
- (75) Boys, S. F.; Foster, J. *Rev. Mod. Phys.* **1960**, *32*, 305.
- (76) Lowdin, P. O. *Phys. Rev.* **1955**, *97*, 1474.
- (77) Bader, R. F. W. *Atoms in Molecules: A Quantum Theory*; Oxford: New York, 1990.

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